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MODELING COGNITIVE DISPOSITIONS OF EDUCATORS FOR EARLY MATHEMATICS EDUCATION

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Research shows that early mathematical abilities are important for future learning and that educators impact the quality of corresponding learning opportunities in kindergarten. However, it is an open question as to how subject-specific cognitive dispositions of educators can be described. Therefore, we adapt a model for mathematics teachers' subject-specific cognition for educators. Besides professional knowledge, the model comprises two components of reflective competence (RC) and action-related competence (AC) that are closely related to professional tasks concerning mathematical learning. Using video-based items, we developed a standardized test. Results on the quality of the measures with N = 112 educators show the usability of the theoretical constructs for empirical investigations.

THEORETICAL BACKGROUND

Many studies indicate the importance of early mathematical abilities for future mathematical performance. Especially, the early number knowledge is considered as crucial for mathematical learning processes at school (e.g., Krajewski, & Schneider, 2009). Although early mathematical learning is often seen as a more implicit and self-regulated process than mathematical learning at school, impressive effects of the quality of the learning opportunities were described (e.g., Sylva et al., 2013). Hereby, the quality of available structures in respect to mathematics (materials, games, etc.) on the one hand and the quality of pedagogical processes (e.g., active mathematical learning support, van Oers, 2009; scaffolding, e.g., Wood et al., 1976) on the other hand are seen to play an important role. Accordingly, educators are an important factor for the quality of mathematical learning environments in kindergarten, as they are responsible for implementing those (for an overview cf. Gasteiger, 2012).

Despite of this consent, it is by far less clear what cognitive characteristics educators should hold in order to deal with the demands of offering these high-quality mathematical learning opportunities for children. Although early education has its own characteristics, we will in the following also use findings on teacher cognition, if analogies seem appropriate. Especially, we will adapt an extended model on teacher cognition comprising knowledge as well as components of competence for educators.

Modeling cognitive dispositions of educators for early mathematics education

Recent research on educators' professional knowledge closely follows the rationale of research on professional knowledge of teachers and differentiates between content knowledge (CK) and pedagogical content knowledge (PCK, Shulman, 1986). CK – in our case mathematical knowledge – is knowledge about early mathematics, its scope, forms, representations, and usage (e.g., Clements & Sarama, 2014; Tsamir et al., 2015). PCK is knowledge about the learning and teaching of mathematics in the early years, including, e.g., knowledge about typical student difficulties, indicators for at-risk students, and adequate learning tasks (e.g., Dunekacke, Jenßen, & Blömeke, 2015; for an international overview cf. Depaepe et al., 2013).

Following the paradigm of expertise research, this professional knowledge is seen as the relevant and specific knowledge base of teaching. Teachers' professional knowledge was accordingly found to predict student learning outcomes, aspects of high-quality instruction, and planning of instruction (Baumert al., 2010; Hill et al., 2008; Dunekacke et al., 2015). However, research cannot explain how professional knowledge of teachers' accounts for these effects in detail. Especially, the relation between knowledge and different practical skills is not extensively investigated.

Teacher knowledge tests focus largely on the measurement of decontextualized, declarative knowledge, so that the demands during testing are very different from practical tasks. Hence, it is an open question if an extended view on teacher cognition that accounts for broader aspects of mathematics-specific teacher cognition could shed additional light on the complex relation between professional knowledge, instructional quality, and student achievement. A first step is to develop a suitable theoretical model including – besides knowledge – cognitive aspects with stronger connections to teaching demands, so that it covers the abilities to utilize this knowledge. Following this idea, we adapted a structure model of subject-specific teacher cognition (Lindmeier, 2011) for our research with educators.

Besides a component of subject-specific professional knowledge (basic knowledge, BK, as CK and PCK), this model uses two components of competence. Competences are here – in a European tradition – context specific and learnable cognitive performance dispositions that allow individuals to cope with certain situations in specific domains (cf., Koeppen et al., 2008), so they can be seen as the cognitive base for practical skills (related to early mathematical education in our case). Lindmeier (2011) differentiates for teachers between practical skills needed to plan, prepare, and post-process instruction and summarizes according cognitive dispositions as reflective competence (RC). In contrast, practical skills to implement high-quality instruction, e.g., to provide active mathematical learning support, are attributed to a different set of cognitive dispositions, called action-related competence (AC). Thus, the differentiation of reflective and action-related competences is theoretically legitimated by two very different groups of demands related to mathematical education: The provision and monitoring of mathematical learning (RC, pre- and post-active), as well as the active scaffolding of mathematical learning (AC, cf. also classroom interactions, Pianta et al., 2005). Educators are then expected to hold specific cognitive dispositions to master these tasks of early mathematical education.

The theoretical model was so far successfully used for research on the cognition of primary and secondary mathematics teachers: Reflective as well as action-related competences proved to be related, but empirically separable components of mathematics teachers' cognition. They were, moreover, separable from professional knowledge (Knievel et al., 2015; Lindmeier, 2011).

For the case of educators' cognition, research findings are less differentiated at the moment. Although there are good reasons to see analogies between the research on teachers' and educators' cognition, there are also some limitations to do so. Before expanding on the relation between professional knowledge and competence, we will introduce the organization of education for educators in our countries, as there are specific differences concerning the role of professional knowledge.

Education of early childhood educators in Germany and in Switzerland

Unlike teachers for primary school (children aged 6+ years), early childhood educators in Germany and Switzerland are not necessarily educated specifically to teach mathematics. In fact, the acquisition of mathematics by children traditionally used to be of little interest, often being part of the educators' education only in relation to the general cognitive development of children. However, the situations in Germany and Switzerland started to diverge, as from 1998 to 2007, the education of pre-school educators (children aged 4-6) in Switzerland was transformed into an academic education (European Qualification Framework, EQF level 6) in parallel to primary teacher education. Hence, early mathematics and its acquisition are studied by the new generation of Swiss educators. In Germany, educators are still mostly educated in non-academic vocational schools (EQF level 4), where the focus lays on the acquisition of practical childcare skills. In sum, this accounts for big differences between the neighbouring countries, so that in 2012 only 3% of the kindergarten educators in Germany have an academic professional education, whereas since a decade all freshly educated Swiss educators have an academic background.

Professional knowledge and action-related competences/practical skills

As explained above, the model we used assumes that the professional knowledge related to teaching mathematics (BK as CK and PCK) is a prerequisite for the two subject-specific competence components (RC, AC) with close relation to professional tasks. In fact, studies with mathematics teachers indicated medium to strong correlations between BK and the competence constructs, especially the relation between BK and RC was found to be substantive (Knievel et al., 2015; Lindmeier, 2011). However, it is questionable if subject-specific professional knowledge of educators plays an equally important role as it does for teachers. At the one hand, subject-specific professional knowledge is not necessarily emphasized in the educators' education, as explained above. Especially, non-academic education should result in lower levels of knowledge about early mathematics education. On the other hand, knowledge concerning early mathematics (especially CK) has a substantial overlap with basic mathematical understanding, so that it is expected to be much less explicitly accessible for educators than professional knowledge concerning mathematics is for teachers. This effect may increase, if the education does not stress this kind of knowledge (cf., Tsamir et al., 2015). Finally, whereas non-academic education is focused on the acquisition of practical skills, academic education is focused on the acquisition of advanced theoretical knowledge and critical thinking. Thus, it is an open question, if the findings of a strong common rooting of AC and RC in BK holds also true for educators.

RESEARCH QUESTIONS AND STUDY DESIGN

The aim of our study was to adapt the three-component model of teacher cognition for kindergarten educators and develop standardized measures for the components in order to allow for a structural investigation of educators cognition. This first study focuses on the viability of the approach. Therefore, we worked on the following research question: (1) Is it feasible to develop valid and reliable instruments to assess the subject-specific components of educators as BK, RC, and AC? (2) Are the resulting measures sensitive to differences expected for groups with known characteristics (discriminant validity)?

Methods

We decided to use a methodological approach that combines standardized assessment using traditional paper-pencil formats with innovative, video-based item formats (similar to Lindmeier, 2011). Video-vignettes of children involved in early mathematics should transport authentic professional demands, enable the educators to mentally engage with those, and thus elicit the target cognitive dispositions. In addition, the instrument was designed to proximally implement the characteristics of the targeted tasks, such as spontaneity (AC) through time constraints. We restricted the contents to the field of numbers and operations due to their importance for mathematical development. The assessment instrument was administered in small groups (up to 10 educators) together with a short questionnaire on personal background (gender, age, education, experience with early mathematics).

Sample

The pilot study is based on the data of 112 participants from Switzerland ($N = 82$) and Germany ($N = 30$). All participants were active kindergarten educators and their age ranged between 20 and 59 years ($M = 35.71$; $SD = 10.58$). The majority of the participants was female (93.8%) and a subsample of 59 (52.7%) educators had a academic education. Our convenience sample did not represent the differences in respect to the academic background according to the framing conditions, as we aimed at a balancing of the contrasting groups and not nationally representative samples (academic track Germany: 56.7%; Switzerland: 51.2%). Of course, as a result of the recent changes, academic education is confounded with age in our sample.

Instruments

In order to operationalize the adaption of the model of Lindmeier (2011), it is necessary to describe the mathematics-specific target tasks of the educators' work and implement them in the assessment instrument. For the assessment of action-related competence (AC), the tasks arise when educators interact in pedagogical situations. Therefore, crucial abilities are to implement mathematical learning environments, provide active learning support, but also to identify and productively use mathematical learning opportunities for a child's development. A sample item targeting at the latter is:

> *Item „set the table":* In the video, two children set the table with miniature doll's dishes and silverware and notice that things are missing. The children say the number of plates and knives available. There are not enough knives and big plates. At the end, the children decide to use smaller plates. After the video, the educators are prompted: "The children already found out that there are things missing. Please ask them a question that converts the situation into a mathematical learning opportunity!"

The educators' open answers were coded according to partial credit scores with 0 credits for general, non-mathematical answers (e.g., "Can you tell me what you noticed?"). Answers that were mathematics-specific, but not transcending what children already noticed were credited with 1 (e.g., "How many plates do you have? How many knives?"). Full credits of 2 were given to mathematics-specific answers, that pick up children's actions and go beyond them (e.g., "You already noticed, that there are things missing. How many plates and knives do you need so that all children have a full setting?"). Overall, seven items for measuring AC were developed.

For the assessment of reflective competence (RC), the range of demands arises from the provision and monitoring of mathematical learning. There are, on the one hand, the preparing tasks to plan, organize, and prepare mathematical learning opportunities. On the other hand, educators are expected to assess and document children's mathematical development, including the task to diagnose specific learning difficulties in the domain. A sample item therefore is given in the following:

> *Item „counting up to 40":* In the video, a 6-year old counts up to 40 and shows characteristic difficulties when crossing the 10s: She falters and cannot name correctly the multiples of 10. Off screen, the educator scaffolds the student when she hesitates at a multiple of 10 by giving the correct name and encouraging: 'What comes next?' The child then autonomously counts on up to the next multiple of 10, but leaves out numbers with repeating digits (22, 33). After the video, the educators are prompted: "The child shows two systematic difficulties. What are they?"

The educators' open answers were coded according to a score with 0 credits for wrong answers or superficial answers not pointing to a mathematical error (e.g., „The child struggles with pronunciation."). Each mathematics-specific difficulty that was identified was scored with 1, so that a maximum of 2 credits could be reached. Altogether, ten items for measuring RC were developed.

Finally, we conceptualized basic professional mathematical knowledge for educators (BK) as knowledge about early mathematics as well as knowledge about the teaching and learning of early mathematics. We did not differentiate between subject knowledge (CK) and pedagogical subject knowledge (PCK) in this study, as both kinds of knowledge were found to be heavily related in studies with teachers and time limitations forced us to lay the measurement focus on the new competence components. Thus, we refrained from using a separate CK test, but integrated CK together with PCK as BK test. For example, we asked the educators to produce different representations for quantities like 8 (CK). PCK items required, e.g., knowledge about the average counting abilities of preschoolers ("Which of the following abilities are usually shown by preschoolers? Counting backwards from 10, 12, or 20"). Altogether, seven items for measuring BK were developed.

Two trained persons scored all answers according to a manual. The mean interrater-reliability was Cohen's $\kappa = .65$ ($\kappa = .50$-$.80$) and considered sufficient in view of the mostly open item formats.

RESULTS

To answer the first research question, we were examined the reliability of the scales. The 24 items showed an internal consistency of Cronbach's $\alpha = 0.88$ ($r_{it} = .23$-$.61$) so that they could be seen as representing a single construct. However, the three subscales showed internal consistencies of comparable sizes ($\alpha_{AC} = 0.77$, $r_{it} = .42$-$.56$; $\alpha_{RC} = 0.78$, $r_{it} = .23$-$.59$; $\alpha_{BK} = 0.71$, $r_{it} = .30$-$.55$). By extrapolating the values according to the length of the overall test (24 items), the reliability of the subscales was estimated to be slightly better than the reliability of the overall scale (Spearman-Brown, $\alpha_{AC24} = 0.92$, $\alpha_{RC24} = 0.89$, $\alpha_{BK24} = 0.89$). Thus, the use of subscales according to the theoretical model can also be justified. A preliminary confirmatory factor analysis indicates also a good fit of a three-dimensional model in line with the theoretical considerations and confirms the correlational findings.

The manifest correlations between the subscales were calculated in order to estimate the strength of relations between knowledge and competence components. A stronger relation between BK and RC ($r = .70$, $p < .01$) was found in comparison to the relations between BK and AC ($r = .53$, $p < .01$) and between AC and RC ($r = .55$, $p < .01$). Hence, the correlational patterns are in line with findings for teachers, indicating a strong common rooting of the competence components in knowledge with yet separable competence components. Reflective competence is stronger associated with knowledge than action-related competence.

	M (SD)			Effect size
Scale	Academic (n = 59)	Non-Academic (n = 53)	t(110)	Cohen's d
AC	9.15 (2.85)	6.06 (2.95)	-5.649*	1.07
RC	12.27 (2.55)	8.17 (3.25)	-7.657*	1.45
BK	9.71 (2.63)	6.49 (2.86)	-6.199*	1.18
ALL	30.95 (6.38)	20.66 (7.10)	-8.078*	1.53

Table 1: Means, standard deviations and results of t-test for differences between the groups of educators with academic vs. non-academic education (* $p < .01$)

The discriminant validity of the scales was examined by comparing two groups of educators (known-groups method, Table 1). The instrument was able to model the expected group differences for the subscales (AC, RC, BK) and for the test in total (ALL). Differences were found to be significant between educators with academic and non-academic background. Thereby, educators with an academic degree achieved higher scores. Effect sizes showed that all differences could be interpreted as large effects.

DISCUSSION

In this study, we adapted a model of teacher subject-specific cognition for the use with kindergarten educators. Besides professional knowledge for early mathematics education (BK), two components of competence closely related to the professional demands of preparing and post-processing mathematical learning (RC) and scaffolding mathematical learning (AC) were introduced. An according measurement instrument for the use in standardized assessment was developed. In order to elicit the targeted competences, our partly video-vignettes based method required the educators to act as-if in practical situations.

The answers of 112 active German and Swiss kindergarten educators were used to investigate the quality of the instrument based on 24 items. The analyses show satisfactory results for the internal consistency of the complete instrument, as well as for the intended subscales. In line with findings from research on teacher cognition, professional knowledge can be seen as an important base for the components of competence. Indeed, there are indications that knowledge is stronger associated to reflective competences than action-related competences. Additional analyses will be conducted with data from a more comprehensive study. We further investigated if our measures are sensitive in respect to expected differences of known groups. In line with the expectations, kindergarten educators with academic education outperform those without an academic background.

Although we can speak of a successful test development, there are also limitations to our study. First, our instrument is at the moment limited to a single, yet especially important mathematical content area (numbers and operations). Second, our instrument uses a combined component of professional knowledge (CK and PCK) due to pragmatic decisions (limited testing time, focus on competence components). Finally, due to a significant change of policies, we cannot disentangle effects of practical experience and (non-)academic education.

Nonetheless, this research paves way for the modeling and measuring of educators' cognitive dispositions for early mathematics education. In a following comprehensive study, we seek to investigate in more detail the relation between areas of educators' professional knowledge and competences as well as the predictive validity of our measures for practical skills. The research thus contributes to a better understanding of what kind of preparation educators need to master the challenging demands of high-quality early mathematics education in kindergarten.

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